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13. ABSTRACT (Maximum 200 words)

Significant progress has been made with solution of location problems and in preprocessing and decomposition for discrete optimization. There has also been research on the application of techniques from combinational optimization to nonlinear problems.

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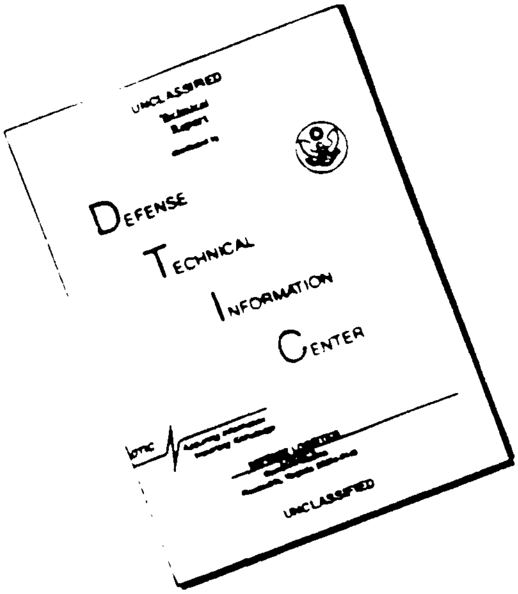

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RUTCOR

FINAL TECHNICAL REPORT

To Air Force Office of Scientific Research

Discrete Applied Mathematics

Grant Number AFOSR 89-0512

Peter L. Hammer
Co-Principal Investigator

Fred S. Roberts
Co-Principal Investigator

October 20, 1993

RUTCOR

Discrete Applied Mathematics:

Grant Number AFOSR 89-0512

FINAL TECHNICAL REPORT

with an emphasis on Research Accomplishments: September 1, 1991 - May 31, 1993

This summary of research accomplishments is organized into essentially the same sections and subsections as is our original proposal. It emphasizes research accomplishments in the last period of the project, September 1, 1991 to May 31, 1993. The reader is referred to the annual technical reports for research accomplishments of earlier years. Papers referred to by number are listed below in the list of publications prepared under the grant during the period September 1, 1991 to May 31, 1993. Papers referred to with authors' names and year are listed at the end of the section.

1. Graph Theory and its Applications

Our research in graph theory has been closely tied to applications. The applications we have considered involve primarily questions of communications and transportation and basic problems in operations research such as scheduling, maintenance, and assignment problems. The specific mathematical questions are divided into two areas, in the following subsections.

1.1. Graph Coloring and Stability

Much current work in graph theory is concerned with the related problems of finding optimal graph colorings and finding the largest stable set in a graph. Problems of graph coloring and stability have been a major area of emphasis for us.

Graph colorings have a wide variety of applications in scheduling, fleet maintenance, traffic phasing, etc. (For a discussion of the applications of graph coloring, see Roberts [1984,1991a].) We have been working on some fundamental problems of graph coloring. In particular, we have studied *list colorings* of graphs. In many practical coloring problems, a choice of color to assign is restricted. A set or list of possible colors to be assigned to a vertex is specified, and we seek a graph coloring so that the color assigned to a vertex is chosen from its list. List colorings arise for instance in channel assignment problems when we specify possible acceptable channels. We have also studied the variant on list colorings that arises when we seek a graph coloring so that the color assigned to a given vertex must not belong to a set of disallowed colors. Erdős, Rubin and Taylor [1979] introduced

the idea of considering when a graph G can be list-colored for every assignment of lists of k colors in each list. If G can always be list colored for every such assignment, we say that G is *k-choosable*. Following Brown, et al. [1990], we say that G is *(j,m)-amenable* if whenever subsets $R(x)$ of $\{1,2, \ldots,m\}$ of size j are assigned to vertices x of G, there is an ordinary graph coloring using colors in $\{1,2,\ldots,m\}$ so that the color assigned to x does not belong to set $R(x)$. Motivated by the relationship between k-choosability and (j,m)-amenability, we say that graph G satisfies property $P(k,J)$ with $0 \leq J \leq \infty$ if for all $j > 0$, G is $(j,j+k)$-amenable if and only if $j \leq J$. In paper [63], we start with the simple observation that G is k-choosable if and only if it is k-colorable and has property $P(k,\infty)$ and we study the problem of identifying graphs having properties $P(k,J)$.

In related work, in paper [47], we solve a problem posed by Erdős, Rubin, and Taylor [1979] by finding an asymptotic result for the list chromatic number (the smallest k so that a graph is k-choosable) of the random graph $G(n,1/2)$.

We have been studying T-colorings of graphs in connection with frequency assignment problems. In such problems, the vertices of a graph G represent transmitters and an edge between two vertices represents interference. We seek to assign to each vertex or transmitter x a channel $f(x)$ over which x can transmit, and for simplicity we take the channels to be positive integers. The assignment of channels is subject to the restriction that if two transmitters interfere, then the channels assigned to these transmitters cannot be separated by a disallowed distance. To make this more precise, we fix a set T of nonnegative integers and assign channels so that if vertices x and y are joined by an edge of G, then $|f(x)-f(y)|$ is not in T. The assignment f is called a *T-coloring*. Its *span* is max $|f(x)-f(y)|$ over all pairs of vertices x and y. See Roberts [1991b] and Tesman [1989] for a summary of the literature of T-colorings and a statement of some of the fundamental problems. In the thesis [69], we have extended and compiled work in earlier years on *no-hole T-colorings*, T-colorings in which $\cup\{f(x): x \in V(G)\}$ is a set of consecutive integers. Heuristic algorithms for T-colorings developed at NATO by T. Lanfear are based on the idea that in the case $T = \{0,1\}$, if there is a no-hole coloring, then such a coloring will come close in span to the optimal (minimal) span. During a visit to RUTCOR by Lanfear, we discussed with him whether this was in fact the case, and indeed, whether there is even always *some* no-hole coloring which comes close to optimal span. Roberts [1993] showed that there can be no-hole colorings which have spans which are very far from the optimal. He also investigated whether or not there is always a no-hole T-coloring which is close to optimal in span, by studying the unit interval graphs which are the simplest case of the r-unit sphere graphs for which T-coloring is especially interesting in practice. He studied the practically important case $T = \{0,1\}$, and showed that if the number of vertices is more than $2\chi(G)-1$, then there is a no-hole T-coloring and all such colorings attain a span which is within one of being optimal. He also showed that if the number of vertices is less than $2\chi(G)-1$, then there is no no-hole

T-coloring. Thesis [69] handles the case where the number of vertices is $2\chi(G)-1$ and generalizes these results to the case $T = \{0,1,\ldots,r\}$. It also studies the graphs which do not have no-hole colorings with $T = \{0,1\}$, and studies $h(G)$, the smallest number of holes left by a T-coloring of G. We find exact values for $h(G)$ for particular graphs and also relate $h(G)$ to the path-covering number and the Hamiltonian completion number of G.

Other variations of graph coloring that have found important practical applications involve the ***k*-tuple colorings** in which every vertex receives a set of k colors and adjacent vertices must receive disjoint sets. These k-tuple colorings have practical applications in problems involving mobile radio frequency assignment, traffic phasing, vehicle maintenance, and task assignment. (See Opsut and Roberts [1981].) Tesman [1989] introduced the idea of combining k-tuple colorings with T-colorings. In thesis [69], we have compiled and advanced results from earlier years on no-hole k-tuple T-colorings, and generalized the results on no-hole T-colorings described in the previous paragraph.

Griggs and Yeh [1992] introduced a variant of T-coloring which they call $L(2,1)$-labelling. Here, we assign nonnegative integers to the vertices of graph G so that adjacent vertices get numbers at least two apart and vertices at distance two get distinct numbers. In thesis [69] we extend and organize work of earlier years on the ***span*** of an $L(2,1)$-labelling, the difference between the largest and smallest label used, and in particular find the span for chordal graphs and for unit interval graphs (which are special cases of chordal graphs).

Considerations of stability have also played a role in our work in the past year. One particular example of research on stability is the work on the size of a stable or independent set in the Clar graph of a benzenoid system in thesis [75], which is discussed in Section 3.1.

The dual concept to stable set, that of maximum clique, is studied in the paper [41]. Here, we present two algorithms for finding maximum cliques in dense graphs.

In Section 2.3, we discuss our work on iterated roof duality in thesis [73], from which we obtain the exact values of the stability number for the special case of odd K_4-free graphs.

Given a positive integer k, determining whether an arbitrary graph contains a stable set of size at least k is NP-complete. However, there are special classes of graphs for which the ***stability number***, the size of the largest stable set, can be computed in polynomial time. In some cases, boolean methods can suggest graph theoretical procedures for determining the size of the largest stable set. In paper [27], we derive from boolean methods a transformation which, when it can be applied, builds from a graph G a new graph G' with the same stability number and one less vertex. We also describe a class of graphs for which such a transformation

leads to a polynomial algorithm for computing the stability number.

At the June 1991 Advanced Research Institute in Discrete Applied Mathematics (ARIDAM VI) held at RUTCOR under AFOSR support, one of the two featured lecturers was Dominic Welsh. His lectures have been written up and expanded in a 178-page RUTCOR Report [74]. Graph coloring plays a major role in this report, which centers on the study of knots and their applications.

Considerations of graph coloring and stability play a central role in the survey paper [67], which was revised and updated in the past year from an earlier version, and which summarizes a variety of important problems and trends in graph theory, with special emphasis on applications.

1.2. Special Classes of Graphs

Many graph theory problems are extremely difficult when looked at in general, but turn out to be tractable when restricted to a special class of graphs. Hence, research in graph theory has in recent years emphasized the study of rich and interesting special classes of graphs, many of which arise from applications, and for which efficient algorithms can often be found to solve important optimization problems. Our work on special classes of graphs has reflected this point of view.

One of the classes of graphs we have studied is the class of threshold graphs. These graphs were defined by Chvatal and Hammer [1977] and have since been studied intensively (see Golumbic [1980]). Among the applications of threshold graphs are applications to Guttman scaling in measurement theory and to synchronizing parallel processors. A finite graph is called *k-threshold* if there is a number h and a linear k-separator w assigning a real number to each vertex so that for any subset S of vertices, the sum of $w(x)$ for x in S is less than h if and only if the subgraph induced by S does not contain a k-clique. *Threshold graphs* are 2-threshold graphs. An *infinite k-threshold graph* is a graph in which every finite induced subgraph is k-threshold. In paper [28], begun last year, we develop the theory of infinite threshold graphs.

The idea of *universal graph*, a graph which contains as an induced subgraph all graphs from a given class, goes back to Rado [1964]. This idea is considered fundamental in extremal graph theory and has important applications in computer science and engineering. A graph is called *T_n-universal* if it contains every threshold graph with n vertices as an induced subgraph. T_n-universal threshold graphs are of special interest since they are precisely those T_n-universal graphs which do not contain any non-threshold induced subgraph. Minimum T_n-universal graphs, T_n-universal graphs with a minimum number of vertices, are especially interesting because they may contain non-threshold graphs as induced subgraphs. In paper [30], we show that for every $n \geq 3$, there are minimum T_n-universal graphs which are themselves threshold and others which are not. The set of all minimum

T_n-universal graphs is described constructively and the proofs provide a polynomial recursive procedure which determines for any threshold graph G with n vertices and for any minimum T_n-universal threshold graph an induced subgraph isomorphic to G. This work is extended in paper [31].

Threshold graphs are also studied in paper [29]. Here we study the Laplacian spectra, the Laplacian polynomials, and the number of spanning trees of threshold graphs. We give formulas for these parameters in terms of so-called composition sequences of threshold graphs and show that the degree sequence of a threshold graph and the sequence of eigenvalues of its Laplacian matrix are "almost the same." Threshold graphs are shown to be uniquely determined by their spectrum and a polynomial time procedure is given for testing whether a given sequence of numbers is the spectrum of a threshold graph.

Related to threshold graphs are *threshold boolean functions*, functions for which there exists a hyperplane separating their set of true points from their set of false points. In the thesis [73] we present a new efficient algorithm for recognizing threshold boolean functions. We also show a hierarchy of generalizations of regular boolean functions, which are themselves natural generalizations of threshold functions. For any of these functions, if the set of minimal true points is given, then the set of maximal false points can be found in polynomial time. A new way of representing positive boolean functions using disjunctive condensed forms is presented. Several polynomial algorithms whose inputs are disjunctive normal forms (DNF's) are generalized to the case when the inputs are disjunctive condensed forms, which are shorter than DNF's.

An important class of graphs with regard to applications is the class of competition graphs and its variants. A graph G is the *competition graph* of a digraph D (often assumed acyclic) if $V(G) = V(D)$ and there is an edge between vertices x and y in G if and only if there is a vertex a of D so that (x,a) and (y,a) are arcs of D. These graphs, introduced by Joel Cohen in 1968, arise in communications over noisy channels (cf. the confusion graphs of Shannon). They also arise in the channel assignment problem mentioned above, which is concerned with coloring a competition graph. They arise in large-scale computer models of complex systems (see e.g., Greenberg, Lundgren, and Maybee [1981]). They arise in the study of food webs in ecology (see e.g., Cohen [1978].) See the surveys by Raychaudhuri and Roberts [1985] and Lundgren [1989]. One of the central problems about competition graphs is the recognition problem. In general, this is NP-complete. However, it can be reduced to the question of computing a parameter called the *competition number*, which is the smallest number of isolated vertices to add to a graph so that the resulting graph is the competition graph of an acyclic digraph. An important tool in the theory of competition graphs is an old result of Roberts that computes the competition number for connected, triangle-free graphs. We have now succeeded in extending this result to the case of connected graphs with exactly one triangle. (See paper [59].)

Competition graphs have been generalized in various ways. Many of these generalizations have been summarized in the survey by Lundgren [1989]. One of them is the *p-competition graph*, a graph G arising from a digraph D (often assumed to be acyclic) by taking $V(G) = V(D)$ and an edge between vertices x and y in G if and only if there are vertices $a_1, \ldots, a_p$ in D so that (x,a_i), (y,a_i) are arcs of D for $i = 1, \ldots, p$. Paper [57], significantly revised from a version produced in earlier years, introduces the notion of p-competition graph and shows how the well-known results about ordinary competition graphs can be generalized to this setting. Corresponding to competition number is the same idea of *p-competition number*. In paper [58] we obtain the surprising result that the p-competition number can be smaller than the competition number, and in fact arbitrarily smaller.

Of great interest in the past 30 years has been the class of perfect graphs first introduced by Claude Berge. See Golumbic [1980]. We have studied the important class of perfect graphs called interval graphs in connection with clustering problems. In many practical problems of detection, decisionmaking, or pattern recognition, we seek methods for clustering alternatives into groups. Clustering methods aim at finding, within a given set of entities, subsets called clusters which are both homogeneous and well-separated. Clustering methods have been used for solving a variety of problems of interest to the Air Force. For instance, they have been used at MAC in locating (through the OADS model) U.S. hubs at Travis Air Force Base in California and Tinker Air Force Base in Oklahoma; in identifying good points of embarkation in deliberate planning models; in identifying staging areas for medical evacuations; and in identifying hubs for the defense courier system. Clustering methods are also relevant to the analysis of various practical problems of the Air Force which involve large amounts of data. These problems arise in such diverse contexts as early warning systems, detection of enemy positions, remote operations in space, cargo movement, "troubleshooting" in complex electronic systems, and forecasting. We have been working on a number of problems that derive clustering from judgements of closeness. In particular, we have studied the interval graph model in which we start with judgements of closeness, assign to each element being judged a real interval, and take two intervals to overlap if and only if the corresponding elements are judged close. This can be accomplished if and only if the graph whose vertices are the elements and whose edges correspond to closeness defines an *interval graph*. Interval graphs arise in numerous applications, including problems involving scheduling, transportation and communications, computer systems, ecosystems, foundations of computation, genetics, and seriation in archaeology and psychology. Specifically, we have studied interval graphs in connection with no-hole T-colorings; see Section 1.1.

We have also studied *unit interval graphs*, interval graphs where all of the real intervals have the same length, and *n^--graphs*, where the real intervals are replaced by sets of n consecutive integers. The n^--graphs arise in problems of visual

perception, involve perceptual judgements of betweenness and proximity that might be relevant to those made by pilots or radar systems, and go back to the work of Goodman [1951] on perceptual geometry. Roberts [1979] showed that the class of unit interval graphs and the union of the classes of n^--graphs are the same. We have studied the problem of finding the minimum n such that a given unit interval graph is an n^--graph. A linear time algorithm to compute this number in a particular case is given, improving the earlier algorithms by Fine and Harop. An (integer) linear programming formulation has also been obtained. These results are discussed in the paper [70] and the thesis [69].

Let q be a positive integer. A *partial q-coloring* of a graph G is a set of q pairwise disjoint stable sets $S_1, \ldots, S_q$. We define $\alpha_q(G)$ to be the maximum of $|US_i|$ over all partial q-colorings. If $V(G)$ is partitioned into cliques $C_1, C_2, \ldots,$ the corresponding *q-norm* is the sum of the terms $\min\{|C_j|, q\}$. We define $\theta_q(G)$ to be the smallest q-norm over all clique partitions of G. In general, $\alpha_q(G)) \leq \theta_q(G)$. For many graphs, these are equal. If a graph G and all its induced subgraphs have this property, we say that G is *q-perfect*. For $q = 1$, this reduces to the classical concept of perfect graph. In paper [3], we study the graphs which appear to be q-perfect for some values of q. We show for instance that every balanced graph is q-perfect for all $q \geq 1$ and we obtain a characterization of q-perfect graphs for $q \geq 2$ in terms of a linear programming problem.

An important class of graphs, for instance in connection with electronic circuits and VLSI design, is the class of planar graphs. In paper [54], we study graph planarity and related topics. We describe different results on graphs containing or avoiding subdivisions of some special graphs, and in particular, different refinements of Kuratowski's planarity criterion for 3-connected and quasi 4-connected graphs. Some results on non-separating circuits in a graph are presented. Some more refinements of Kuratowski's theorem and graph planarity criteria in terms of non-separating circuits are given for 3-connected and quasi 4-connected graphs. An ear-like decomposition for quasi 4-connected graphs is described similar to that for 3-connected graphs, and is shown to be a very useful tool for investigating graph planarity and some other problems for quasi 4-connected graphs. Refinements of different kinds are given for Whitney's graph planarity criterion. Some results on Dirac's conjecture and Barnette's conjecture are also presented.

Planarity also plays a role in paper [44]. Here, we study planar triangulations and in particular obtain bounds on the bandwidth of such triangulations. The bandwidth is an important concept that is related to channel problems in communications and has been widely studied in graph theory.

Induced subgraphs with a tree structure arise in a variety of problems in telecommunications and design of reliable networks for communication, transportation, and power. In paper [56], we have initiated the study of the problem of optimally packing in a graph

the special tree subgraphs called stars.

Classes of graphs which have particular kinds of orientations have played an important role in graph theory and its applications. From the point of view of moving traffic (both vehicles and information), graphs which have strongly connected orientations are of central importance. It has been known for more than fifty years which graphs have strongly connected orientations. What is interesting to ask is what makes one strongly connected orientation better than another. One can introduce a variety of criteria of efficiency of a strongly connected orientation (see Roberts [1976,1978] for many examples). Unfortunately, for none of the widely studied criteria is the problem of finding the most efficient strongly connected orientation solvable by an efficient algorithm for a general graph. However, for the graphs which arise in practical traffic routing problems, considerable progress has been made. In a series of papers prepared in earlier years, we have studied the grid graphs consisting of north-south streets and east-west avenues, and found optimal strongly connected orientations under two fundamental criteria. Now, in paper [4], we have started to make considerable progress on the same problem for annular cities.

Extremal problems for graphs arise in a variety of important practical applications and often involve useful special classes of graphs. In paper [62] we study a collection of extremal problems for graphs that arose in connection with distributed computing. These questions turned out to be closely related to questions about sphere-packing in graphs.

In a recent series of articles, R. Jamison and S. Olariu developed, starting from an extension of the notion of a cograph, a decomposition theory of graphs into P_4-connected components. It turned out that the algorithmic idea to exploit the unique tree structure of cographs can be generalized to graphs with simple P_4-structure. In paper [43], we show that hamiltonicity and the computation of the path covering number are both easy for P_4-sparse and P_4-extendible graphs, generalizing a result of H.A. Jung.

Special classes of graphs play a central role in the survey paper [67], which, as noted in Section 1.1, was revised and updated in the past year from an earlier version, and which summarizes a variety of important problems and trends in graph theory, with special emphasis on applications.

2. Discrete Optimization

Discrete optimization problems arise in a large variety of vitally important practical scheduling, allocation, planning, and decisionmaking problems. We have been putting a considerable emphasis on such problems in this project.

2.1. Location Problems

Location problems arise whenever a large set of potential sites for placing certain units is available and a selection must be made of the sites to be utilized. Such problems arise naturally in situations like placing warehouses, satellites, communication centers, military units, or emergency services. They are especially important in Air Force problems involving locations of hubs, points of embarkation, staging areas, and other facilities; such problems arise frequently in the network routing questions at MAC or the Air Force Logistics Center. See Hansen, et al. [1987] for a recent survey. We have been studying a variety of location problems and approaches to solving them.

Weber's problem, which consists of locating a single facility in order to minimize the sum of Euclidean distances between its location and those of a given set of users, is the most studied one of continuous location theory. Since its statement in Weber [1909], many generalizations have been considered. One of these is the *multifacility Weber problem* where several facilities are to be simultaneously located in the Euclidean plane in order to minimize the sum of weighted distances between a given set of users and their closest facilities. A second is the *conditional Weber problem* which arises when some facilities are already established, and the problem is to locate a new facility in order to reduce the most the sum of distances from the users to the closest existing or new facility. Paper [17] and thesis [15] are concerned with applications of d.-c. programming to these generalizations of the Weber problem. *D.-c. programming* is a recent technique of global optimization which allows the solution of problems whose objective function and constraints can be expressed as differences of convex functions. We have obtained good computational results for problems with up to a thousand users, twenty existing facilities, and three new facilities.

Paper [18] and thesis [15] are also concerned with aspects of Weber's problem. These publications extend work of last year which was concerned with the case where some weights are positive and some are negative. This case was also shown to be a d.-c. program, reducible to a problem of concave minimization over a convex set. The latter is solved by outer-approximation and vertex enumeration. Moreover, locational constraints can be taken into account by combining the previous algorithm with an enumerative procedure on the set of feasible regions. The major new result is that the algorithm has been extended to solve the case where the obnoxiousness of the facility is assumed to have exponential decay. The paper reports on computational experience with up to 1000 users.

Traditionally in the theory of location problems, one is interested in minimizing or maximizing some objective function and this objective function is assumed a priori. Since there are so many potential objective functions, attention must be paid to how to choose an appropriate function. Recently, Holzman [1990] specified some reasonable conditions that an objective function for solving a location problem should satisfy. He showed that as long as the

network in which the facilities were to be located had a tree structure, then the objective function was uniquely determined by these conditions. Vohra [1990] obtained similar results for trees under other conditions. In paper [64], we have found a simpler result than Vohra's which also holds in a variety of different contexts, in particular which allows both the users and the facilities to either only be at vertices of the network or to be anywhere along the edges.

Networks with tree structures are rather special and so the results of Holzman, Vohra, and paper [64] referred to above are somewhat special. We have investigated more general networks. In earlier years, we had found that a reasonable set of axioms related to Holzman's is self-contradictory for some non-tree networks. In the past year, we have found the startling result that under very general conditions closely related to Holzman's, for all connected networks other than trees, there is no objective function satisfying these conditions. By omitting one of the assumed conditions, one can restate the result in the following way: Under some reasonable conditions which one would like to impose on the objective function in solving a location problem, the solution can be very sensitive to small changes in locations of the users of the facilities. This has important practical implications for implementing the solutions to real location problems. The results are written up in paper [42].

In recent years, a considerable amount of discrete mathematical research has been brought to bear on methods for understanding and improving group decisionmaking. Two of the most useful group decisionmaking procedures are the median procedure and the closely related mean procedure. These procedures are used in aggregating preferences in social choice problems; in voting; in statistical analysis as concepts of centrality; in molecular biology in the process of finding consensus patterns in molecular sequences; and in mathematical taxonomy in finding the consensus in classification problems. In paper [68] we present a general framework for speaking about the median and mean procedures and use it to organize a wide variety of recent results. In particular, we are able to show how various approaches to finding the objective function in location problems on networks, to which we have referred above, fall into this framework.

2.2 Preprocessing and Decomposition

Discrete optimization problems arise frequently in an unmanageable form. One approach is then to transform a given problem after some manipulation into a more structured one or a small number of more structured problems, for which good solution methods exist. Our research effort has given considerable emphasis to such preprocessing and decomposition of discrete optimization problems.

Among the discrete optimization problems we have studied in which decomposition is a theme is a problem of clustering. In Section 1.2, we described the important Air Force applications of

clustering methods. A well-known clustering algorithm, the *k-Means algorithm*, partitions elements (in Euclidean space) into two distinct clusters according to a dissimilarity measure (the Euclidean distance). The algorithm is based on the minimization of a certain functional, through a descent procedure. Although this algorithm runs quite fast, quickly detecting clusters in large data sets, it misclassifies a great number of objects when clusters are quite different in size. We have devised a new algorithm, based on quadratic 0-1 minimization. In this algorithm, we have an important preprocessing step in order to reduce the number of objects, followed by an application of quadratic 0-1 minimization methods on the resulting more tractable problem. This algorithm constructs a much better classification than the k-Means algorithm; in tests, it has repeatedly almost completely retrieved the original clusters which were randomly generated selecting objects from two normal distributions. See paper [6]. Other work on clustering in this project is in the paper [12], which is discussed in Section 3.1, and in the paper [22], which gives algorithms for average linkage divisive hierarchical clustering.

Boolean satisfiability problems are central to combinatorial algorithms, both because they encompass many important combinatorial problems and as the first example of a combinatorial problem which is NP-complete. Let V be a set of n boolean variables and V' the set of boolean complements of these variables. The elements of $L = V \cup V'$ are called *literals*. Given a boolean formula in conjunctive normal form (CNF), the *satisfiability problem* consists of finding a satisfying true/false assignment to the variables or in recognizing that no such assignment exists. A common simplification of this problem consists of assigning the obvious values to the "pure" literals, i.e., to those which appear only in complemented or only in uncomplemented form. In paper [8], we present a linear time algorithm for determining a subset F^* of the variables and a true/false assignment to these variables such that if there exists a satisfying assignment at all, then there exists one with these same values on F^*. The set of literals fixed by this algorithm includes properly, and can be substantially larger than, the set of pure literals.

Much work has been done to establish subclasses of satisfiability problems which are solvable in polynomial time. In paper [7] we associate a real-valued index to each instance of satisfiability and show that, in some sense, this index measures how hard the problem is. We give an algorithm for satisfiability which runs in polynomial time on any instance for which the value of this index is below a certain threshold. In contrast, we show that for instances where the index is larger than the threshold the problem is NP-complete.

As above, let V be a set of n boolean variables and V' the set of boolean complements of these variables. A *partial assignment* is a subset S of literals so that $S \cap S' = \phi$. A *Horn formula* is a boolean formula on these n variables so that for all partial assignments L, $|L \cap V'| \leq 1$. Horn formulae play a prominent role in artificial intelligence and logic programming. Their

importance is due to a large extent to the fact that for such expressions the satisfiability problem is linearly solvable. The class of q-Horn boolean expressions, generalizing quadratic, Horn, and disguised Horn formulae, was introduced in Boros, Crama, and Hammer [1990], in which it is shown that the satisfiability problem corresponding to such an expression is solvable in time linear in the size of the expression. The recognition of such formulae, however, was based on the solution of a linear programming problem, and had therefore a much higher complexity. In paper [11], a combinatorial algorithm is presented for recognising q-Horn formulae in linear time. Similar results are described in the thesis [73].

In paper [33], we introduce the concept of a ***Horn function***, a boolean function which admits a representation by a Horn formula. Horn functions arise in a variety of applications, including in particular the analysis of production rule knowledge bases of propositional expert systems. In the paper we observe that the irredundant prime disjunctive normal forms (DNF's) of any Horn function are Horn. We reduce the study of the irredundant prime DNF's of Horn functions to the study of the irredundant prime DNF's of pure Horn functions. This reduction is achieved by proving that every prime irredundant DNF of a Horn function consists of a prime irredundant DNF of its "pure Horn component," and of a "positive restriction" of the function. We provide a constructive characterization of all the positive restrictions, and present an efficient algorithm for decomposing any Horn function into its pure Horn component and its positive restriction. Finally, we reduce in quadratic time the problem of minimizing the number of terms in DNF of a Horn function to the same problem for its pure Horn component.

When a given signal can be interpreted as being the result of a variety of causes and a small number of tests have to be created to identify the exact cause of the signal, we have a typical instance of a ***multiple conclusion logic situation***. Examples of such situations occur in medical decisionmaking, in "troubleshooting" in complex systems including networks and electronic and mechanical systems, in searching and seeking in hazardous or nuclear or chemically toxic environments, in detecting enemy positions in remote operations in space or underseas, and so on. We have worked on a number of approaches to such problems. Specifically, we have studied the multiple conclusion logic problem involving optimal compression of knowledge bases in expert systems. In paper [32], we formalize the compression of knowledge bases as the problem of boolean function minimization and investigate this problem for the widely used class of propositional Horn clause bases. We use the concept of Horn function and consider the special class of quasi-acyclic Horn functions, which properly includes the two practically significant classes of quadratic and of acyclic functions. We develop a cubic time procedure for recognizing the quasi-acyclicity of a function given by a Horn CNF. A graph-based algorithm is proposed for the quadratic time minimization of quasi-acyclic Horn functions.

In paper [34], we continue the investigation of the problem of

optimal compression of propositional Horn production rule knowledge bases. The standard approach to this problem, consisting of the removal of redundant rules from a knowledge base, leads to an "irredundant" but not necessarily optimal knowledge base. We prove here that the number of rules in any irredundant Horn knowledge base involving n propositional variables is at most n-1 times the minimum possible number of rules. In order to formalize the optimal compression problem, we define a boolean function of a knowledge base as being the function whose set of true points is the set of models of the knowledge base. In this way, the optimal compression of production rule knowledge bases becomes a problem of boolean function minimization. In this paper we prove that the minimization of Horn functions (i.e., boolean functions associated to Horn knowledge bases) is NP-complete.

Paper [35] deals with the minimization of quasi-acyclic Horn functions, the class of which properly includes the two practically significant classes of quadratic and of acyclic functions, as noted above. A procedure is developed for recognizing in quadratic time the quasi-acyclicity of a function given by a Horn CNF, and a graph-based algorithm is proposed for the quadratic time minimization of quasi-acyclic Horn functions.

Other work on the multiple conclusion logic problem is contained in paper [10]. This paper addresses the problem of predicting the value of a function on the basis of discrete observational data that are incomplete in two senses: Only certain arguments of the function are observed, and the function value is observed only for certain combinations of values of these arguments. We address the problem under a monotonicity condition that is natural in many applications, and we discuss applications to tax auditing, medicine, and real estate valuation. In particular, we display a special class of problems for which the best monotone prediction can be found in polynomial time.

Other preprocessing methods have been used in our study of various types of mathematical programming problems. For instance, indefinite quadratic programs with quadratic constraints can be reduced to bilinear programs with bilinear constraints by duplication of variables, as described in Section 2.4. In paper [36], described in that section, we study such reductions in which the number of additional variables is minimum or the number of complicating variables, i.e., variables to be fixed in order to obtain a linear program in the resulting bilinear program, is minimum.

In many situations, multiple decisionmakers with divergent objectives intervene in decisions to be made. The simplest such case, in which there are only two decisionmakers, has long been studied in game theory. If there is some asymmetry between the decisionmakers, in that one of them, called the *leader*, makes his decisions first, anticipating the reaction of the other one, called the *follower*, and cooperation is ruled out a priori, we have what is called a *Stackelberg game*. Adding joint constraints on the strategies of the leader and the follower makes the model more

realistic and leads to *bilevel programming*, a topic which has attracted much attention recently. Building on work begun in earlier years, we have studied linear bilevel programming (see paper [37]). We have proposed a new branch-and-bound algorithm for linear bilevel programming. We have used necessary optimality conditions expressed in terms of tightness of the follower's constraints to fathom or simplify subproblems, branch and obtain penalties similar to those used in mixed-integer programming. Our computational results compare favorably to those of previous methods and we have been able to solve problems with up to 150 constraints, 250 variables controlled by the leader, and 150 variables controlled by the follower.

Preprocessing methods are often useful in the study of scheduling problems. We have studied a scheduling problem in which n tasks are to be scheduled on a single processor without preemption, the processing time of each task is given, and each task is only allowed to start for execution at a time-unit chosen from a given set of possible starting times. We ask whether or not there is a feasible schedule for all tasks. Applications of this problem arise for example in the area of computation in real-time environments and in the manufacturing of printed circuit boards. Earlier results by Nakajima and Hakimi, Keil, and Crama and Spieksma have shown that for different values of the processing times and the number of possible starting times, the complexity of the problem differs widely. In paper [21] we consider a major remaining case, and show that if all the tasks have common processing time $r = 2$ and there are $k = 3$ possible starting times, then the problem is NP-complete. This result establishes a sharp boundary between NP-complete and polynomially solvable versions of this problem with respect to the parameters r and k.

2.3. Approximation

A major theme in discrete mathematics in recent years has been to find methods for approximating solutions to problems and to find exact solutions by successive approximations. The approximation problem has been an important focus of our efforts.

Flow problems on networks belong to the most studied problems of mathematical programming. They have numerous applications in practice since highway, rail, electrical, communication and many other physical networks have widespread use. We have used a network flow based algorithm in thesis [73] to study the NP-hard problem of minimizing *quadratic pseudo-boolean functions*, i.e., quadratic real-valued polynomials whose variables take only the values 0 and 1. The network flow algorithm finds a lower bound for the minimum. This approach gives the same lower bounds as others (such as the well-known roof duality of Hammer, Hansen, and Simeone [1984]), but provides a faster algorithm to compute the lower bound. We also note how the max-flow approach can also quickly identify the optimal values of a subset of variables and report computational results.

Also in thesis [73], we provide better bounds than roof duality

by presenting an approach called iterated roof duality. We show that iterated roof duality applied to a class of quadratic pseudo-boolean functions which are naturally associated to graphs provides the exact values of the stability number for the special case of odd K_4-free graphs.

Roof duality also plays a role in the paper [14]. We consider there the ***weighted maximum 2-satisfiability problem***: given a quadratic formula in CNF, let a positive weight be associated with each clause and find a truth assignment maximizing the total weight of the clauses that are satisfied. This problem is equivalent to the problem of finding the minimum z^* of a quadratic posiform. We describe a polynomial time algorithm for computing a lower bound on z^*. The algorithm consists of a finite sequence of elementary boolean operations of two types: fusions $(x + x' = 1)$ and exchanges $(x + x'y = y + y'x)$. Our main result is that the bound obtained by this method is equivalent to the roof duality bound, which is known to be computable by linear programming. Furthermore, one can check in polynomial time whether such bound coincides with z^*. If not, one can obtain strictly sharper lower bounds making use of two further elementary boolean operations called condensation and consensus.

In paper [16], we discuss the vertex enumeration problem for polytopes, both in an off-line and on-line manner. We describe this work in detail in Section 4.2. Among its many applications, also described there, is that a solution to the off-line problem allows one to solve the approximation problem of finding all near-optimal solutions to a linear program.

In paper [26], we give a brief and elementary proof of a result of Hoffman [1952] about approximate solutions to systems of linear inequalities. We improve upon the earlier results in several ways. First, we obtain a simple proof which relies only on linear programming duality; second, we obtain geometric and algebraic representations of the bounds that are determined explicitly in terms of the underlying matrix; and finally, our bounds with respect to general norms are sharper than those obtained previously.

Networks with weights on the arcs occur in many important Air Force applications. We have been interested in such networks in which the arcs represent activities, each activity has a certain required time, the vertices represent stages, and each stage is required to be reached at the same time. In particular, these kinds of problems arise in transportation networks with travel times on the arcs and where synchronized arrivals are required; in PERT networks of activities where all activities leading to a given stage must end at the same time; and in parallel machines in which the arcs correspond to different processors and where each processor requires all of its input signals to arrive at the same time (pipelining the data flow). The problem is to add idle times for each activity so that synchronization can be achieved at each stage in such a way as to minimize the total idle time. In paper [9], we have presented the first polynomial time solution for the problem of finding the idle time assignment which minimizes the total amount of

idle time in a network. We have also shown that the problem of minimizing the maximum idle time can be solved in polynomial time, while the problem of minimizing the number of activities with positive delay from some ideal completion time is NP-complete.

Branch and bound algorithms for integer programming problems typically employ bounds derived from well-known relaxations such as the Lagrangian, surrogate, or composite relaxations. Although the bounds derived from these relaxations are stronger than the bound obtained from the linear programming relaxation (LPR), in the case of multidimensional knapsack problems, i.e., integer programming problems with nonnegative objective function and constraint coefficients, the improvement in the bound that can be realized using these relaxations is limited. In paper [19] we show that the improvement in the quality of the bound using any of these relaxations cannot exceed the magnitude of the largest coefficient in the objective function, nor can it exceed one-half of the optimal objective function value of LPR. This implies, for example, that for those problem classes in which all of the objective function coefficients are equal to 1, the bound derived from the surrogate relaxation cannot be better than the bound obtained by simply rounding the LPR bound. Awareness of these properties is important in the development of algorithms, since this class of problems subsumes many well-known integer programming problems.

2.4. Applications of Combinatorial Optimization to Nonlinear Problems

In operations research, one makes the distinction between algorithms designed to find a local optimum and algorithms designed to find the global optimum. The vast majority of nonlinear programming algorithms belong to the first category, but increasing attention is being devoted to the latter one. We have found that many of the ideas underlying algorithms for combinatorial optimization can be transposed to the field of global optimization and we have been exploring this idea.

In Section 2.1, we have described our work on d.-c. programming and its applications to Weber's problem of location theory and various generalizations of that problem. As we remarked there, d.-c. programming is a recent technique of global optimization that allows the solution of problems whose objective function and constraints can be expressed as differences of convex functions. These results are described in papers [17] and [18] and thesis [15].

Several pieces of work in this project have been concerned with quadratic programs. Quadratic optimization problems arise in a variety of important applications including selection of R&D projects, selection of petroleum leases upon which to bid, selection of items to be included in any volume-limited or weight-limited space, and the selection of routes to be served by a commercial or military carrier. Paper [38] studies global minimization of quadratic functions subject to constraints of a box type.

Building on work begun in the previous year, we have been concerned with reduction of global optimization problems to bilinear programs. A *bilinear program* involves minimization of a bilinear function subject to bilinear constraints. In paper [36], we concentrate on the special case of quadratic programs. General quadratic programs appear to be very difficult to solve exactly. However, reduction of such programs to bilinear programs by duplication of variables appears to be a fruitful approach. We have studied the problems of finding such reductions in which either the number of additional variables is minimum or the number of complicating variables, i.e., variables to be fixed in order to obtain a linear program in the resulting bilinear program, is minimum. In paper [36], begun last year, these two problems are shown to be equivalent to a maximum bipartite subgraph and a maximum stable set problem respectively in a graph associated with the quadratic program. Non-polynomial but practically efficient algorithms for both reductions are thus obtained. Reduction of more general global optimization problems than quadratic programs to bilinear programs is also discussed.

Paper [39] deals with interval arithmetic and decomposition applied to univariate global optimization. Paper [40] also deals with univariate global optimization. Here, we find a nice formulation of Taylor's expansion.

3. Combinatorial Structures and their Applications

Combinatorial structures such as matroids, graphs, block designs, and partially ordered sets have a wide variety of applications in practical problems. We have investigated a variety of such structures and their applications in this project.

3.1. Useful Combinatorial Structures

While our work at RUTCOR on combinatorial structures has emphasized graphs (see Section 1), we have also investigated a variety of structures including certain kinds of hypergraphs, partial orders, matchings, matrices, and geometric configurations.

Many combinatorial structures arise in the study of geometry. The *Borsuk conjecture* concerns such a structure; it states that every subset of Euclidean d-space of unit diameter can be covered by $d+1$ sets each of diameter strictly less than 1. In the book by Croft, Falconer, and Guy [1991], this conjecture is called "one of the most famous unsolved problems of geometry." We have recently solved this problem by showing that the conjecture is wildly incorrect. The result is written up in paper [48].

Special classes of matrices are often of interest in combinatorics. Let A be an $n \times n$ matrix of real numbers. A is called a *Z-matrix* if all its off-diagonal entries are nonpositive and an *F-matrix* (a *Ky Fan N-matrix*) if $A = \alpha I - B$ with $B \geq 0$

and $\lambda < \alpha < \rho(B)$, where I is the identity matrix, $\rho(B)$ is the spectral radius of B, and λ is the maximum of the spectral radii of all principal submatrices of B of order $n-1$. We show in paper [66] that if A is a Z-matrix, then it is an F-matrix if and only if a certain linear complementarity problem has exactly two solutions for any positive q and at most two solutions for any other q.

In paper [65], we investigate *M-matrices*, real matrices A of the form $sI - B$, where $s > \rho(B)$. We investigate the similarity between positive definite matrices and M-matrices. Some well-known inequalities for positive definite matrices are shown to be true for M-matrices. The main results are analogues of the Minkowski, Bergstrom, and Fan inequalities for the difference of two M-matrices.

Partial orders are a very useful combinatorial structure. In Section 4.1 we discuss our applications of randomization methods in paper [51] to the problem of sorting a partially ordered set by comparisons. This problem arises in a variety of practical problems, described in Section 4.1.

One structure of considerable interest in decisionmaking has been the hypergraph. A hypergraph is called *intersecting* if no two edges are disjoint. In the mid-1980's, Füredi and Seymour conjectured that if F is an intersecting hypergraph on n vertices, then there is a set of n pairs of vertices such that each edge of F contains one of the pairs. The conjecture is a stronger version of a special case of the Borsuk conjecture discussed above. The fractional version of the Füredi-Seymour conjecture was proved by Füredi and Seymour and then in a stronger form by Alon and Seymour. We have now shown in paper [49] that the Füredi-Seymour conjecture is false. The counterexample is obtained by random methods.

A fundamental result of Pippenger, based on a well-known theorem of Frankl and Rödl, is that for any k-uniform, D-regular, pseudosimple hypergraph H on n vertices, $\rho(H) \sim n/k$, where $\rho(H)$ is the minimum size of a hypergraph $H' \subset H$ whose edges cover the vertex set of H. In paper [50], we obtain a substantial extension of Pippenger's Theorem which essentially describes how far the assumption of pseudosimplicity may be relaxed.

An important class of problems in the theory of computational complexity involves determining the minimum size of circuits for computing various classes of boolean functions. In paper [45] we consider threshold circuits whose basic element is a threshold gate, a multiple input-single output unit described by a linear function of its inputs, which outputs 1 on a certain input if the value of the linear function exceeds 1. Little has been proven about the limitations of such circuits. In this work, we establish a trade-off between the size and depth of these circuits. In particular, our result provides the first superlinear lower bound for an explicit function (in this case, the parity function) on the size of a bounded depth threshold circuit that computes it.

Matchings (pairwise disjoint sets of edges) are combinatorial structures in graphs that are of great importance in a variety of applications, including assignment problems for jobs, tasks, and storage, and specifically in assigning pilots to aircraft. We have been concerned with a variety of problems involving matchings. The thesis [1] is concerned with the stable matching problem. In this problem, agents must be matched in pairs while having preferences over their potential mates. The goal is to find a matching where no matchable pair of agents prefer each other to their outcome under the matching. Such a *matching* is called *stable*. Stable matchings have a variety of practical uses, in particular in assignment of residents to hospitals. Stable matchings are a generalization of the *stable marriage problem*, the special case where each agent is labelled as either a man or woman and each matchable pair consists of a man and a woman. Gale and Shapley showed that a stable marriage problem always has a stable matching. We show that their algorithm can be interpreted as a dual simplex method. We characterize the graphs for which all associated stable matching problems have stable matchings. Irving obtained the first polynomial algorithm for non-bipartite problems that finds a stable matching or determines that none exists. We show that linear programming, combined with a simple graph search procedure, yields an alternative polynomial algorithm.

We have also been concerned with the special class of matchings called perfect matchings, with an emphasis on their applications to chemistry. The thesis [75] is concerned with perfect matchings in benzenoid systems. It organizes and extends work we have been doing on this subject in past years. A *benzenoid system* is defined to be a finite connected subgraph of the infinite hexagonal lattice which has no nonhexagonal interior faces or cut edges. Such graphs are also of considerable importance in communication networks, specifically in mobile radio communication. We have focused on perfect matchings of such graphs. Benzenoid systems, which are the skeletons of benzenoid hydrocarbons, are important for both chemists and combinatorialists. See Cyvin and Gutman [1988]. Vast experimental data show that many chemical properties of benzenoid hydrocarbons, such as stability and color, can be explained by the topological properties and structure of the corresponding benzenoid systems. We have developed two algorithms to determine a *Kekulé structure* (or a perfect matching) of a benzenoid system. One of them has linear computational complexity and the other corrects the peeling algorithm of Gutman and Cyvin.

Also in thesis [75] we present a mixed integer programming model to determine *Clar formulas* of benzenoid systems or, in other words, maximum sets of mutually resonant hexagons. It turns out that mixed integer programming and, in practice, linear programming, allows us to solve efficiently this problem even for very large molecules (pericondensed benzenoids with several hundred hexagons). We also develop a method to calculate various upper bounds on the *Clar number*, i.e., the number of mutually resonant hexagons in a Clar formula.

Also in thesis [75], we prove Cyvin and Gutman's conjecture that a normal benzenoid system (or a benzenoid system without fixed bonds) with h (> 1) hexagons can be constructed from a normal benzenoid system with h-1 hexagons. This conjecture is the basis of a sieve method to enumerate all normal benzenoid systems with a number of Kekulé structures less than or equal to a given number.

While not exactly a combinatorial structure, the concept of topological space is sometimes very useful in studying combinatorial problems. We have found this to be the case in the study of a problem arising from distributed computing. Much of the theory of distributed computing is concerned with problems of coordinating a set of processors. Various problems have been proposed to model simple coordination tasks. A famous result of Abu Amara-Loui and Herlihy (based on a previous result of Fischer-Lynch-Patterson) says that, in the standard asynchronous shared-memory with atomic read/write, a very basic coordination task called "consensus" is impossible to do with absolute reliability. Chaudhuri conjectured that the same impossibility result extends to a weaker coordination task called "k-set consensus." In paper [72] we prove this conjecture by showing that the structure of the problem can be represented by a topological space, and the impossibility result follows from a fixed point theorem on that space.

Another widely used combinatorial structure is the hypercube, which is very important in network design, computer architecture, coding, etc. Paper [71] is concerned with the interesting and potentially useful problems that arise from slicing hypercubes.

A *switchbox* is a rectangular grid of horizontal tracks and vertical columns. Such rectangular grids are interesting combinatorial objects. A *net* is a collection of boundary points of such a switchbox and a *switchbox routing problem* is a set of pairwise disjoint nets. The solution of such a routing problem is the realization of the nets as pairwise vertex disjoint subgraphs (usually Steiner trees) of the planar grid graph so that each subgraph connects the boundary points of the net. In paper [13], we consider the gradually more and more complex problems of switchbox routing called single row routing and channel routing, and the gradually less and less restrictive models (involving different assumptions about the realization) called 1-layer, Manhattan, unconstrained 2-layer, and multi-layer. The single row routing problem can always be solved in the Manhattan model and the channel routing problem can always be solved in the unconstrained 2-layer model, in fact both in linear time. In paper [13] we show that the general switchbox routing problem is solvable in the multilayer model, also in linear time.

Partitions of integers are among the most widely studied combinatorial structures. A partition of a set N of n distinct numbers is called *nested* if there do not exist four numbers $a < b < c < d$ in N such that a and c are in one part while b and d are in another. A partition is called a *p-partition* if the number of parts is specified at p and a *shape-partition* if the sizes of the p parts are also specified. There are exponentially many

p-partitions but only polynomially many nested p-partitions. Recently, Boros and Hammer showed that under certain partition-cost functions, an optimal p-partition is always nested. In paper [12], we give a general condition on the cost structure for which an optimal shape-partition is always nested. We illustrate applications of our results to some clustering problems.

In paper [25], we study oriented matroids and generalize the following theorem: Consider a polytope P and a facet F_0 of P, and let H denote the hyperplanes spanned by F_0. Let d denote the diameter of the coskeleton of P. For each facet choose a defining inequality and let these sets of inequalities F be partitioned by the distance of the corresponding facets F_0 in the coskeleton of P into $F = \cup_{i=1}^{d} F_i$. Let P_i' denote the polyhedron defined by the inequalities F_i and set $P_i = P_i' \cup H$. Then for all $i < d$, $P_i \subset P_{i+1}$. This is known as the Nested Cones Theorem.

3.2. Random Discrete Structures and their Applications

An increasing theme in discrete mathematical research is to investigate random discrete structures of various kinds. The reason for the emphasis on random structures is in part because of their connections to probabilistic algorithms (see Section 4.1) and in part because of their relevance in formulating models for applied problems. Moreover, sometimes a probabilistic approach can lead to useful results about inherently non-probabilistic problems. We have worked on a number of problems in this area, some concerning behavior of random objects, and others whose solutions seem likely to require a probabilistic approach.

Counting spanning trees is a fundamental problem in enumerative combinatorics. The conventional approach uses determinants, and explicit formulas have been found for several classes of graphs, undirected and directed. In paper [23], begun last year, we concentrate on certain families of digraphs that evolve naturally from the study of certain longstanding unsolved problems in queueing theory, i.e., involving networks of queues with finite buffers. We derive a new method for counting the number of trees rooted at any given node, and this in turn yields the steady state queue length probabilities for queues of finite length. To demonstrate the method, we have derived explicit formulas for the number of trees, and hence, the queue length probabilities, for the case of two queues in tandem with smallest buffer size at most 2.

In many practical problems of communications, transportation, power distribution, etc., one is dealing with a network which is vulnerable because some of its components are subject to failure. The theory of *network reliability* has been developed to deal with this problem, and it has taken much of its motivation from the literature of command, control, and communications. (See for instance Bracken [1983].) There is a long literature concerning alternative definitions of network reliability and means of computing the reliability of a network under different models of the random failure of components. (See for instance the monograph by

Colbourn [1987] and the volume edited by Hwang, Monma and Roberts [1991].) In paper [55], we have found lower and upper bounds on network reliability using operations on graphs that increase or decrease reliability.

The thesis [60] is also concerned with reliability problems, in particular with reliability optimization. The approaches and results are described in Section 4.3.

Most military decisionmaking takes place in a climate of uncertainty and risk. Theories of risk have a long history in the literature of decision theory. Continuing work begun in the previous year, we have presented in paper [24] a discussion of the prediction in the economic theory of choice that, under appropriate restrictions on the shape of the objective function, all risk-averters reduce their activities when they are shifted from a certainty environment to a risky one with the same mean. When mean preserving 'marginal' changes in risk - instead of "global" ones - are considered, risk aversion is no longer sufficient to predict the decisionmaker's response even for simple problems with a linear objective function. Hence, further restrictions have to be imposed in order to find again the kind of result obtained in the "no risk to risk" case. The search for well-behaved results has led economists to pay attention almost exclusively to the utility function. We have made considerable progress by considering the other ingredient of the decision problem, the distribution function of the random variable. We build on earlier work of Meyer and Ormiston and others by extending the class of changes in the distribution that give unambiguous comparative statics results. We do so by comparing distributions with stochastic dominance in the tails.

Randomness is an important aspect of the theory of sorting of information. Our work on this theory, in paper [51], is described in Section 4.1.

Randomness has also been a central theme in the work in paper [47], described in Section 1.1, in which we solve a problem posed by Erdős, Rubin, and Taylor [1979] by finding an asymptotic result for the list chromatic number of the random graph $G(n,1/2)$.

Random methods are used in paper [49] to obtain a counterexample to a conjecture of Füredi and Seymour about intersecting hypergraphs, as described in Section 3.1.

An old problem, which comes up in various contexts, is to estimate the probability P_n that a random $n \times n$ matrix of +1's and -1's is singular. In paper [53], we show that there is a positive constant ϵ for which $P_n < (1-\epsilon)^n$. This is a considerable improvement on the best previous bound $P_n = O(1/\sqrt{n})$ given by Komlos in 1977.

Matchings are important in many contexts (see discussion in Section 3.1, for example). We have studied random matchings in papers [46, 52]. In [46], we associate a random variable $\xi(G)$

with a graph, where $\xi(G)$ is the size of a matching chosen uniformly at random from the set of matchings of G. We show that the mean and variance of $\xi(G)$ are remarkably well-determined just by degree and number of vertices. In [52], we study a sequence of simple graphs G_n and the sequence of probabilities $p_k(G_n)$ that the n^{th} graph in the sequence has ξ equal to k. We give necessary and sufficient conditions for the distribution $\{p_k(G_n)\}$ to be asymptotically normal. In particular, our results imply asymptotic normality for any sequence of regular graphs or graphs with perfect matchings.

4. Efficient Algorithms for Discrete Problems

One of the major changes in discrete mathematics in the 1970's and 1980's has been the strong emphasis on algorithms. We have reflected this emphasis throughout the project by studying algorithms for a variety of discrete problems. We have emphasized several themes which we see as increasingly important and which are described in this section.

4.1. Probability and Algorithms

It has long been known that many algorithms which can be bad in their worst cases are very good in an "average" case. This has led to increased interest in analysis of algorithms over random instances of problems. Here, the inputs are drawn from a known distribution and we seek algorithms with good average case behavior. Recent studies of the average case behavior of the simplex algorithm are important examples of what we have in mind. Probabilistic ideas enter into the development of efficient algorithms in another way as well. Namely, sometimes if we allow a machine to make some random choices, we obtain an algorithm -- a random algorithm -- which is very effective at solving a problem. We have studied the interaction between probability and algorithms in both of these ways.

Partial orders are a very useful combinatorial structure. In a variety of practical decisionmaking problems, one is given data in the form of a partial order and is asked to extend it to a linear order. This occurs for example when individual preferences are expressed and a linear order of alternatives is required. It also occurs in activity networks in project planning, where for some activities x and y, we are told that x must be completed before y can be started; and in designing a glossary when we wish to define each word before it is used. The linear extension problem is also important in many problems of sorting in computer science. In work begun last year, we have made in paper [51] an important breakthrough on the problem of finding the best way to sort a partially ordered set by comparisons, a problem originally considered by Fredman in the 1970's. Sorting trivially requires at least $\log(e(P))$ comparisons for a partially ordered set P with $e(P)$ linear extensions or possible sorts. Fredman showed in 1976 that $\log(e(P)) + 2n$ comparisons suffice, where $n = |P|$. Kahn and Saks showed in 1984 that it can be done in $O(\log(e(P))$

comparisons, by proving the existence of a "good" comparison, meaning one which more or less splits the extensions. In both cases, finding the desired comparisons seems quite intractable. We have now shown that one can sort a partially ordered set P using O(log(e(P)) comparisons and find the comparisons in deterministic polynomial time. This is a major development, which makes the results useful in practice. The results take a completely new approach to the problem, based on entropy of the comparability graph and convex minimization via the well-known ellipsoid algorithm. The results present a nice example of the usefulness of recent powerful randomized methods for volume computation (Dyer, Frieze, and Kannan [1989], Karzanov and Khachiyan [1991]) based on rapidly mixing Markov chains, and the trick is to allow randomization.

In Section 4.2, we describe our work in paper [5] on on-line algorithms for server problems. Randomized algorithms play a central role.

The *witness finding problem* for a family R of subsets of a finite set X is to produce a subset S of X that intersects each member of R. Finding deterministic algorithms for such problems seems to be an important step in eliminating randomness from algorithms. In paper [61] we have developed a solution for the case that X is a product set and R is the set of combinatorial rectangles of relative measure bounded below by a fixed constant.

4.2. On-Line Methods

There is increasing emphasis in practical problems to find solution algorithms which are on-line in the sense that one is forced to make choices at the time data becomes available, rather than after having the entire problem spelled out. A large effort, the U.S. Transcom/DARPA Planning and Scheduling Initiative, is devoted to the development of a highly interactive strategic mobility modelling tool which will allow an Air Force user to get detailed information about many aspects of the Air Force transportation systems and make on-line decisions. The emphasis on such a modelling tool underlines the importance of on-line methods for current Air Force problems.

A general approach to on-line problems is to think of them as sequential decisionmaking problems. There are two points of view: (a) formulate a probabilistic model of the future and minimize the expected cost of future decisions; (b) compare an on-line decision strategy to the optimal off-line algorithm, one that works with complete knowledge of the future. The first is the approach taken for instance in the theory of Markov decision models. We have emphasized the second approach.

In an on-line computational problem, the algorithm determines a sequence of actions in response to a growing sequence of "requests" from the environment, with the goal being to minimize some cost function. An important measure of quality of an on-line algorithm, which has become standard in the literature, is the competitive ratio, which measures, roughly, the worst-case over all input

sequences, of the ratio of the cost incurred by the algorithm to the cost that an off-line algorithm could have paid on that sequence. Much effort has been expended to determine, for a wide range of problems, how small a competitive ratio can be obtained. This computational setting is one in which randomized algorithms can be shown to have a provable advantage over deterministic ones. In paper [5] we have made important progress in determining the extent of this advantage for a class of problems called *server problems*. Here, the algorithm controls a set of k servers, which sit in some metric space. At each time step, the environment makes a request, which is represented by a point in the metric space. The algorithm must serve the request by moving a server to the point (if none is there already). The algorithm is charged a cost equal to the total distance moved. Such problems arise naturally in many of the Air Force server problems arising at MAC, specifically in problems where planes (servers) from k hubs are requested to deliver cargo or passengers to different destinations. We have developed a new analytic tool for analyzing the optimal competitive ratio attainable by a randomized algorithm for server problems. This tool provides a way to decompose a server problem into smaller ones and relate the competitive ratio of the given problem to the competitive ratios of the smaller ones. As a consequence, we have determined upper and lower bounds and useful new algorithms for server problems.

The *off-line vertex enumeration problem* for polytopes consists of determining all vertices of a given polytope P. The *on-line vertex enumeration problem* consists of determining all vertices of $P \cap H$, where H is a given half-space, assuming vertices of P are known. In paper [16], begun last year, we discuss both of these problems. The off-line problem can be solved by searching the adjacency graph of P, pivoting iteratively from a tableau corresponding to a vertex of P to a tableau corresponding to an adjacent unexplored vertex. Improving on a result of Dyer, we first show that this can be done in $O(mnv)$ time, where m is the number of facets, n is the dimension, and v is the number of vertices of P. Adjacent vertices of a vertex for which a tableau is known can be determined using only two columns of that tableau each time, i.e., by partial pivoting. We discuss and compare several algorithms from the literature that use partial pivoting. We propose a new algorithm in which the number of vertices for which the whole tableau must be built is reduced. This is attained by using hash coding to detect adjacencies between such vertices and their neighbors as well as between pairs of these neighbors. Computational results are reported. Finally we note that the on-line vertex enumeration problem can be stated as an off-line problem after elimination of a variable. This strategy is compared theoretically and/or empirically with recent algorithms for on-line vertex enumeration given in the literature.

4.3. Heuristics

As more and more problems are shown to be difficult, for instance by proving them to be NP-complete, there is coming to be an increasing emphasis on heuristic solutions. Heuristic algorithms are especially important in practice where there are many problems

involving hundreds, thousands, even tens of thousands of variables. In such a case, we would like to elaborate a heuristic algorithm capable (in most cases) of very rapidly finding (approximate) solutions to large problems.

Many of the algorithms we have been developing are heuristic in nature. They have been described elsewhere in this report. In this section, we mention just a few additional examples of our work where heuristics play a very central role.

Automated manufacturing systems play a vital role in increasing productivity in manufacturing. The most recent among these systems are characterized by extremely high levels of speed and accuracy. Unfortunately, the complexity of the planning problems associated with these systems usually increases in direct relation to their technological sophistication. We have made important progress on several planning and scheduling problems related to automated manufacturing. In paper [20], we have also successfully addressed several types of tool management problems. A central issue in tool management for flexible manufacturing systems consists in deciding how to sequence the parts to be produced and what tools to allocate to the machines in order to minimize the number of tool setups (which disrupt the production flow). We have developed a "column generation" approach which has allowed us to solve to optimality much larger instances of certain tool-generating problems than those previously handled in the literature.

In Section 1.2, we described the importance of clustering methods for a variety of problems of interest to the Air Force and described some of our work on clustering. Here, we mention a quite different application of clustering that arises in problems of planning and scheduling in automated manufacturing. Automated manufacturing systems play a vital role in increasing productivity in manufacturing. The most recent among these systems are characterized by extremely high levels of speed and accuracy. Unfortunately, the complexity of the planning problems associated with these systems usually increases in direct relation to their technological sophistication. We have made important progress on several planning and scheduling problems related to automated manufacturing. Specifically, we have successfully addressed several types of tool management problems. A central issue in tool management for flexible manufacturing systems consists in deciding how to sequence the parts to be produced and what tools to allocate to the machines in order to minimize the number of tool setups (which disrupt the production flow). We have developed a "column generation" approach which has allowed us to solve to optimality much larger instances of certain tool-generating problems than those previously handled in the literature. The results are written up in paper [20].

In Section 3.2 we have discussed some of our work on reliability of networks. Large reliability optimization problems in series-parallel or in complex systems are difficult to solve. A tabu search heuristic is provided in thesis [60] to determine the optimum or near-optimum number of redundant components in each

subsystem. For the case of series-parallel systems redundancy optimization can be expressed as a zero-one linear program with generalized upper bounding (GUB) structure. We extend the algorithm of Dantzig and Van Slyke for linear programming with GUB structure to the case of zero-one variables. The resulting algorithm allows us to solve rapidly problems with several hundred variables. In the thesis [60], we also study two-terminal and all-terminal reliability of general networks, with independent probabilities of failure on the arcs, using Boole-Bonferroni inequalities (cf. Prekopa [1988]).

The oil pipeline network problem (closely related to the one-terminal TELPAK problem in telecommunications) consists of determining the layout and diameters of the pipes of a network that connect a given set of offshore platforms and onshore wells to a port. We have designed a specialized implicit enumeration algorithm, extending the one developed for reliability optimization and mentioned in the previous paragraph. This is described in thesis [60]. We also define two new types of valid inequalities exploiting geometric properties of feasible solutions. Finding those of the first type amounts to solving multiple choice knapsack problems and those of the second type to enumerating spanning trees. Algorithms and implementation for counting and enumerating spanning trees of graphs are included in [60]. A large application is made with simulated data from south Gabon oil field.

Assignment of the modules of a parallel program to processors of a multiple computer system has been studied by Bokhari. He proposed algorithms to solve optimally the following problems: (i) partition chain-structured parallel or pipeline programs over chain-connected systems; (ii) partition multiple chain-structured parallel or pipelined programs over single-host multiple-satellite systems; (iii) partition multiple arbitrarily structured serial programs over single-host multiple-satellite systems; (iv) partition single tree-structured parallel or pipelined programs over single-host multiple identical satellite systems. In thesis [60] we provide algorithms with reduced computational complexity for all four problems.

When dealing with quantitative analysis in an economic model it does not suffice to know that the model has an equilibrium. This led H. Scarf to describe an algorithmic approach to find such equilibria. In paper [2] we report on computational experience with an implementation of three algorithms for the general economic equilibrium problem. As a result we conclude that the projection algorithm for variational inequalities increases the size of solvable models by a factor of 5-10 in comparison with the classical homotopy method. As a third approach we implemented a simulated annealing heuristic which might be suitable to estimate equilibria for large models.

4.4. Approximate Algorithms

As we observed in Section 2.3, a major theme in discrete mathematical research in recent years has been to find methods for

approximating solutions to problems and to find exact solutions by successive approximations. Our work in this area is summarized in that section.

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Lectures Delivered and Miscellaneous Honors

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Miscellaneous Honors

Fred Roberts' work on traffic flow was written up in a featured article in the *Chronicle of Higher Education* in January 1992. A copy of the article is attached.

Lectures Delivered

Peter L. Hammer

"On a Polynomially Solvable Class of Satisfiability Problems," at ORSA/TIMS joint National Meeting, Anaheim, CA, November 1991.

"Network Flows and Roof Duality for Quadratic 0-1 Programming," at ORSA/TIMS joint National Meeting, Anaheim, CA, November 1991.

"Reducibility of Set Covering to a Knapsack Problem," plenary talk at Computer Science and Operations Research Conference, Williamsburg, Virginia, January 1992.

"A Complexity Index for Satisfiability Problems," plenary talk at Conference on Integer Programming and Combinatorial Optimization, Carnegie-Mellon University, Pittsburgh, PA, May 1992.

"Max-cut via Cycle Packings," plenary talk at EURO XII, Helsinki, Finland June 1992.

"Satisfiability of Quadratic Horn Forms," plenary talk at International Conference on Graphs and Optimization," Grimentz, Switzerland, August 1992.

"Boolean Methods in Discrete Optimization," plenary talk at First Russian-Italian Conference on Methods and Applications of Mathematical Programming, Italy, September 1992.

"Generalized Pure Literal Rule," ORSA/TIMS National Meeting, San Francisco, November 1992.

"Recognition of Q-Horn Formulae in Linear Time," ORSA/TIMS National Meeting, San Francisco, November 1992.

"Horn Function Minimization for Query Speedup in Propositional Expert Systems," ORSA/TIMS National Meeting, San Francisco, November

1992.

"Unconstrained Quadratic 0-1 Programming: A Computational Study," Symposium on Applied Mathematical Programming and Modeling, APMOD, Budapest, January 1993.

"Graphs and Boolean Functions," University of Puerto Rico, January 1993.

"Clique Separation and Universal Threshold Graphs," invited talk at Cambridge Combinatorial Conference in Honor of Paul Erdös, Trinity College, Cambridge, England, March 1993.

"Network Balancing Problems Arising in VLSI Design," ORSA/TIMS National Meeting, Chicago, IL, May 1993.

"Iterated Roof Duality and Multicommodity Flows for Quadratic 0-1 Programming," ORSA/TIMS National Meeting, Chicago, IL, May 1993.

"A Depth-first-search for Boolean Satisfiability: A Computational Study," ORSA/TIMS National Meeting, Chicago, IL, May 1993.

"On the Essential and Redundant Rules in Horn Knowledge Bases," ORSA/TIMS National Meeting, Chicago, IL, May 1993.

Fred S. Roberts

"Elementary, Sub-Fibonacci, Regular, Van Lier, and other Interesting Sequences," plenary talk at Sixth Clemson Conference on Discrete Mathematics, Clemson, South Carolina, October 1991.

"The One Way Street Problem," Pi Mu Epsilon Talk, Seton Hall University, South Orange, NJ, November 1991.

"Mathematics, Traffic Flow, and the Environment," invited presentation at Joint Policy Board in Mathematics Press Briefing on Mathematics and the Environment, American Mathematical Society National Meeting, Baltimore, MD, January 1992.

"An Impossibility Result in Axiomatic Location Theory," plenary talk at European Chapter on Combinatorial Optimization, Annual Meeting, Graz, Austria, April 1992.

Four-lecture series on "Applications of Discrete Mathematics" at William Paterson College, Wayne, NJ, March-April 1992.

"Consensus Functions and Patterns in Molecular Sequences," plenary talk at Fourth Stony Brook Biomathematics Conference, Stony Brook, NY, May 1992.

"An Impossibility Result in Axiomatic Location Theory," plenary talk at International Conference on Graph Theory and its Applications, Kalamazoo, MI, June 1992.

Five lectures on Graph Theory and its Applications to DIMACS Leadership Workshop on Discrete Mathematics, New Brunswick, NJ, July 1992. Titles of Lectures:
"The One-Way Street Problem."
"Applications of Graph Coloring."
"Applications of Eulerian Chains and Paths."
"Applications of Simple Counting Rules I."
"Applications of Simple Counting Rules II."

"On the Median Procedure," invited talk at 4th International Conference on Information Processing and Management of Uncertainty, Palma de Mallorca, Spain, July 1992.

"On the Meaningfulness of Ordinal Comparisons for General Order Relational Systems," plenary talk at European Mathematical Psychology Group, Annual Meeting, Brussels, Belgium, July 1992.

Four invited plenary talks on Measurement Theory, its Applications, Meaningless Statements, and Dimensional Analysis at Conference on Mathematical Systems Underlying Axiomatic Measurement Theories, Irvine, CA, July 1992. Titles of talks:
"The Representational Approach to Measurement."
"Measurement Theory and Social Choice."
"Applications of the Theory of Meaninfulness."
"On the Possible Merging Functions."

"The One-Way Street Problem," plenary talk, Fall Meeting, Mathematical Association of America, Seaway Section, Cornell University, Ithaca, New York, November 1992.

"On Greedy and No-Hole Graph Coloring," colloquium talk, University of Tennessee, Knoxville, December 1992.

"The One-Way Street Problem," DIMACS Conference on Discrete Math in the Schools, New Brunswick, NJ, January 1993.

"Innovative Curricula in the Mathematical Sciences for the 90's and Beyond," Conference on Graduate Programs in the Mathematical Sciences for the 90's and Beyond, Clemson University, Clemson, South Carolina, April 1993.

"Sturdy Networks," invited talk at American Mathematical Society Meeting, Washington, DC, April 1993.

"Sturdy Networks," plenary talk, conference on Graphs and Matrices, Boulder, Colorado, May 1993.

"Meaningless Statements," colloquium talk, University of Pittsburgh, Pittsburgh, PA, May 1993.

<u>Hernan Abeledo</u>

"Stable Matchings and Polyhedral Combinatorics," Combinatorics Seminar, The George Washington University, Washington, DC, October

1992.

Endre Boros

"On a Polynomially Solvable Class of SAT Problems," invited talk at ORSA/TIMS Joint National Meeting, Anaheim, CA, November 1991.

"A Complexity Index for SAT," plenary talk at Second International Symposium on Artificial Intelligence and Mathematics, Ft. Lauderdale, FL, January 1992.

"Reducibility of Set-Covering to a Knapsack Problem," invited talk at the ORSA Computer Science Technical Section Conference, Williamsburg, VA, January 1992.

"A Complexity Index for SAT Problems," presentation at the 2nd Conference on Integer Programming and Combinatorial Optimization, Carnegie-Mellon University, Pittsburgh, PA, May 1992.

"On a Complexity Index for SAT Problems," invited lecture at First Hungarian-American Combinatorial Optimization Workshop, Cornell University, Ithaca, NY, October 1992.

"On Hard and Easy SAT Problems," plenary lecture at 8th Clemson Mini-conference on Discrete Mathematics, Clemson University, Clemson, SC, October 1992.

"Generalized Pure Literal Rule," ORSA/TIMS National Meeting, San Francisco, November 1992.

"Recognition of Q-Horn Formulae in Linear Time," ORSA/TIMS National Meeting, San Francisco, November 1992.

"A Complexity Index for SAT," and other lectures, Symposium on Applied Mathematical Programming and Modeling, APMOD, Budapest, January 1993.

"An Exact Algorithm for SAT with Computational Results," invited lecture at DIMACS workshop on Solving Hard Combinatorial Optimization Problems," DIMACS, Rutgers University, March-April 1993.

"Unconstrained 0-1 Programming with Applications in Image Processing and Clustering," invited lecture at DIMACS workshop on Partitioning Data Sets: With Applications to Psychology, Vision, and Target Tracking, DIMACS, Rutgers University, April 1993.

"Quadratic 0-1 Programming Applied to QAP," invited lecture at DIMACS workshop on Quadratic Assignment and Related Problems," DIMACS, Rutgers University, May 1993.

"Network Balancing Problems Arising in VLSI Design," ORSA/TIMS National Meeting, Chicago, IL. May 1993.

"Iterated Roof Duality and Multicommodity Flows for Quadratic 0-1 Programming, ORSA/TIMS National Meeting, Chicago, IL, May 1993.

"A Depth-first-search for Boolean Satisfiability: A Computational Study," ORSA/TIMS National Meeting, Chicago, IL, May 1993.

Pey-chun Chen

"Vertex Enumeration with Hash Coding and the Neighborhood Problem," at ORSA/TIMS joint National Meeting, Anaheim, CA, November 1991.

"Generalized Weber Problems and D.C. Programming, at ORSA/TIMS joint National Meeting, Orlando, FL, April 1992.

"Vertex Enumeration Methods and Continuous Location Theory," GERAD, Ecole des Hautes Etudes Commerciales, Montreal, Canada, July 1992.

Yves Crama

"The Polytope of Block Diagonal Matrices," invited talk at Symposium on Applied Mathematical Programming and Modeling, Budapest, January 1993.

"Throughput Rate Optimization in the Automated Assembly of Printed Circuit Boards," invited lecture at Conference on Advanced Production Techniques, Liege, Belgium, February 1993.

Four invited lectures on "Combinatorial Optimization Models for Production Planning in Automated Manufacturing," Seminar of the Swiss Operations Research Society, Grimentz, Switzerland, March 1993.

"A Complexity Index for Satisfiability Problems," invited lecture at NGB Seminar on Combinatorial Optimization, Utrecht, Netherlands, May 1993.

Pierre Hansen

"From the Median to the Generalized Center," plenary lecture at Journees Franco-Suisse de Recherche Operationnelle, Paris, September 1991.

"On Weber's Problem," plenary lecture at XVIth SOR Meeting, Trier, Germany, October 1991.

"Partial Pivoting in Vertex Enumeration," plenary talk at Second Workshop on Integer Programming and Combinatorial Optimization, Carnegie-Mellon University, Pittsburgh, PA, May 1992.

"Clustering Algorithms," ONR Workshop on Discrete Structures in Classification, Herndon, Virginia, May 1992.

"New Branch and Bound Rules for Linear Bilevel Programming," plenary

talk at International Conference on Graphs and Optimization, Grimentz, Switzerland, August 1992.

"Constrained Nonlinear 0-1 Programming," conference inaugural lecture, XXIVth SOBRAPO Conference, Salvador, Brazil, November 1992.

"A Comparison of Algorithms for the Maximum Clique Problem," conference inaugural lecture, Symposium on Applied Mathematical Programming and Modeling, APMOD, Budapest, January 1993.

Jeff Kahn

"A Problem of Erdős and Lovasz and other Stuff," colloquium talk, Rutgers University, New Brunswick, NJ, February 1992.

"Asymptotics of Packing, Covering and Coloring Problems," seminar talk, University of Michigan, Ann Arbor, April 1992.

"Asymptotics of Packing, Covering and Coloring Problems," colloquium talk, Carnegie-Mellon University, Pittsburgh, PA, April 1992.

"A Semirandom Method for Hypergraph Problems," 4 hours of lectures, Hebrew University, Jerusalem, June 1992.

"Geometric Aspects of Sorting Partially Ordered Sets," Technion, Haifa, June 1992.

"Singularity Probabilities for Random {+/-1}-matrices," Technion, Haifa, June 1992.

"Asymptotically Good List Colorings," Technion, Haifa, June 1992.

"Asymptotically Good List Colorings," invited talk in special session on probabilistic combinatorics at American Mathematical Society/London Mathematical Society meeting, Cambridge, England, June 1992.

"A Counterexample to Borsuk's Conjecture," invited talk in special session on discrete geometry and convexity at American Mathematical Society/London Mathematical Society meeting, Cambridge, England, July 1992.

"Asymptotics of Packing and Coloring Problems," invited talk at American-Hungarian Workshop on Combinatorial Optimization, Cornell University, Ithaca, NY, October 1992.

"A Counterexample to Borsuk's Conjecture," Colloquium, MIT, Cambridge, MA, October 1992.

"Asymptotics of Packing and Coloring Problems," invited talk in Symposium on Trends in Discrete Mathematics, Bielefeld, Germany, October 1992.

"A Counterexample to Borsuk's Conjecture," invited talk at

Kombinatorik, Oberwolfach, Germany, November 1992.

"Geometric Aspects of Sorting Partially Ordered Sets," invited talk, Geometry Day, Courant Institute, New York, November 1992.

"Borsuk's Conjecture is False," Geometry seminar, Courant Institute, New York, November 1992.

"Entropy and Sorting," invited talk, Entropy Day, Rutgers University, New Brunswick, NJ, November 1992.

"On the Probability that a Random ± 1 Matrix is Singular," Yale University, New Haven, March 1993.

"Random Matchings," colloquium, University of Illinois, Champaign-Urbana, April 1993.

"Random and Asymptotic Aspects of Matching Theory," series of four lectures, Technion, Haifa, Israel, May 1993.

"Random Matchings," invited talk at Jerusalem Combinatorics, Hebrew University, Jerusalem, May 1993.

"A Normal Law for Matchings," invited talk in special session on probabilistic methods, American Mathematical Society meeting, DeKalb, IL, May 1993.

<u>Alexander Kelmans</u>

"Packing in a Graph Subgraphs of Special Type," at Southeastern International Conference on Combinatorics, Graph Theory, and Computing, Boca Raton, FL, February 1992.

"A Generalization of the Edmonds-Gallai Theorem on Graphs," invited seminar talk, MIT, Cambridge, MA, March 1992.

"Reliable Networks from Non-reliable Elements," invited seminar talk, Boston University, Boston, MA, March 1992.

"Non-Hamiltonian Graphs and Barnette's Conjecture," invited seminar talk, McMaster University, Hamilton, Ontario, Canada, May 1992.

"Some Optimal Packing Problems for a Graph and a New Class of Matroids," invited seminar talk, University of Waterloo, Waterloo, Ontario, Canada, May 1992.

"Random Graphs and Network Reliability," invited seminar talk, York University, Toronto, Ontario, Canada, May 1992.

"Tutte's Conjecture on Bipartite, Cubic, and 3-Connected Graphs," invited seminar talk, University of Waterloo, Waterloo, Ontario, Canada, May 1992.

"Coding of Spanning Trees of the Extended Graphs," invited seminar

talk, MIT, Cambridge, MA, May 1992.

"Coding of Spanning Trees of the Extended Graphs," invited seminar talk, Northeastern University, Boston, MA, May 1992.

"Universal Threshold Graphs," Advanced Research Institute in Discrete Applied Mathematics, RUTCOR, June 1992.

"Recent Results on Planarity," invited talked at Seventh International Conference on Graph Theory, Combinatorics, Algorithms, and Applications, June 1992.

"Graph Planarity and Related Topics," plenary talk, Conference on Graph Minors, Seattle, Washington, June-July 1992.

"A Generalization of Matching Theory," invited talk at DIMACS workshop on Disjoint Path Problems," DIMACS, Rutgers University, November 1992.

"Some Constructions of Non-Hamiltonian Graphs," University of Puerto Rico, May 1993.

<u>Alexander Kogan</u>

"Logic Minimization," plenary talk at conference on Computer Science and Operations Research: New Developments in their Interfaces, Williamsburg, VA, January 1992.

"Horn Functions and their DNF's," invited plenary talk at DIMACS Forum, Princeton, NJ, April 1992.

"Horn Function Minimization and Knowledge Compression in Production Rule Bases," at Seventh Advanced Research Institute in Discrete Applied Mathematics, RUTCOR, Rutgers University, New Brunswick, NJ, June 1992.

"Horn Function Minimization for Query Speedup in Propositional Expert Systems," ORSA/TIMS National Meeting, San Francisco, November 1992.

"Applications of Horn Functions," invited talk at Conference on Expert Systems in Accounting, Auditing, and Business Telecommunications, Graduate School of Management, Rutgers University, Newark, NJ, November 1992.

"A Depth-first-search for Boolean Satisfiability: A Computational Study," ORSA/TIMS National Meeting, Chicago, IL, May 1993.

<u>Keh-wei Lih</u>

"Solving Mixed-Integer Programming Problems with GUB Constraints," ORSA/TIMS joint National Meeting, Anaheim, CA, November 1991.

"A Mathematical Approach to Oil Pipeline Design," ORSA/TIMS joint National Meeting, Orlando, FL, April 1992.

Uriel Rothblum

"Approximations to Solutions of Linear Inequalities," SIAM meeting, Minneapolis, MN, October 1991.

"Formulation of Linear Problems and Solution by a Universal Machine," invited talk at Matrix Workshop, Institute for Mathematics and its Applications, Minneapolis, MN, November 1991.

"Stable Marriages and Linear Inequalities," Faculty of Industrial Engineering and Management, Technion, Haifa, December 1991.

"The Optimality of Clustering and Monotone Optimal Assemblies," invited talk, Workshop on Discrete Structures in Classification, Herndon, VA, May 1992.

"Stable Marriages and Polyhedral Combinatorics," RUTCOR colloquium, Rutgers University, September 1992.

"Linear Problems -- Formulation and Solution," RUTCOR, September 1992.

"Stable Matchings and Polyhedral Combinatorics," Operations Research Department, Stanford University, Stanford, CA, February 1993.

"Stable Matchings and Polyhedral Combinatorics," School of Business, UCLA, Westwood, CA, February 1993.

"Linear Problems: Formulation and Solution," International Linear Algebra Society, Pensacola, FL, May 1993.

Denise Sakai

"Unit Interval Graphs, n-graphs and Uniform n-Extensions," 23rd Southeastern International Conference on Combinatorics, Graph Theory and Computing, [illegible] Raton, FL, February 1992.

"Generalized [illegible] Colorings," seminar, Seton Hall University, South Orange, NJ, [illegible] 1992.

"Generalized Graph Colorings," seminar, City College of Staten Island, Staten Island, NY, February 1992.

"Generalized Graph Colorings," Naval Postgraduate School, Monterey, CA, February 1992.

"Generalized Graph Colorings and the Channel Assignment Problem," seminar, Marist College, Poughkeepsie, NY, March 1992.

"Generalized Graph Colorings," seminar, Montclair State College, Upper Montclair, NJ, March 1992.

"Generalized Graph Colorings and Intersection Assignments," seminar, University of Chicago, Chicago, IL, March 1992.

Michael Saks

"The Topology of Distributed Knowledge," Hebrew University, Jerusalem, January 1993.

Maolin Zheng

"On Clar Number and Clar Formula," Seventh Advanced Research Institute in Discrete Applied Mathematics, RUTCOR, June 1992.

"Perfect Matchings in Benzenoid Systems," GERAD, H.E.C., Montreal, September 1992.

Participants in RUTCOR Project on "Discrete Applied Mathematics"

September 1, 1991 - May 31, 1993

Faculty

Peter Hammer (Principal Investigator)

Fred Roberts (Principal Investigator)

Endre Boros

Vasek Chvatal

Pierre Hansen

Jeff Kahn

Uriel Rothblum

Michael Saks

Postdoctoral Fellows

Alexander Kelmans

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Jianmin Long

Jianmin Miao

Dale Peterson

Denise Sakai

Li Sheng

Xiarong Sun

Shaoji Xu

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Ronald Graham, AT&T Bell Laboratories

Laszlo Lovasz, Eotvos University and Princeton University